Solve the equations below by factoring and using the square root property.

**Completing the Square**

It's possible to take any quadratic equation, create a perfect square trinomial, and solve it in a similar way. This method is called _completing the square_.

1. **Rewrite** as $ax^2 + bx + c$.
2. **Divide** both sides by '$a'$ so it becomes $x^2 + bx = c$.
3. **Complete the square** by taking half of $b$, square it, and add it to both sides of the equation.
4. **Factor** the perfect square trinomial.
5. **Take the square root** of both sides. This will create two cases because a square root has both a positive and negative value.
6. **Solve** both equations. **Simplify** all rational and complex answers.

**Directions:** Solve each quadratic equation below by completing the square.

3. $x^2 - 18x + 56 = 0$
   - $x^2 - 18x + 84 = -56$
   - $(x - 9)^2 = 16$
   - $x = 9 + 4$ or $x = 9 - 4$
   - $x = 13$ or $x = 5$

4. $2x^2 - 16x = -30$
   - $x^2 - 8x + 15 = -15$
   - $(x - 4)^2 = 1$
   - $x = 4 + 1$ or $x = 4 - 1$
   - $x = 5$ or $x = 3$
3.4 Completing the Square and Complex Numbers

5. \( \frac{4x^2 - 8x}{4} = \frac{-3}{4} \)
   \[ x^2 - 2x = -\frac{3}{4} \]
   \[ x^2 - 2x + 1 = -\frac{3}{4} + 1 \]
   \[ (x - 1)^2 = \frac{1}{4} \]
   \[ x - 1 = \pm \frac{1}{2} \]
   \[ x = \frac{3}{2} \quad x = \frac{1}{2} \]

6. \( 3x^2 + 10x + 8 = 0 \)
   \[ \frac{-10 \pm \sqrt{100 - 48}}{6} \]
   \[ x^2 + \frac{16}{3}x + \frac{8}{3} = 0 \]
   \[ x^2 + \frac{16}{3}x + \frac{16}{9} = \frac{8}{3} - \frac{16}{9} \]
   \[ (x + \frac{8}{3})^2 = \frac{4}{9} \]
   \[ x + \frac{8}{3} = \pm \frac{2}{3} \]
   \[ x = -\frac{2}{3} \quad x = -2 \]

7. \( x^2 + 16x - 21 = -5 \)
   \[ x^2 + 16x = 16 \]
   \[ (x + 8)^2 = 80 \]
   \[ x + 8 = \pm 4\sqrt{5} \]
   \[ x = -8 \pm 4\sqrt{5} \]

8. \( 3x^2 - 30x = 69 \)
   \[ (x - 5)^2 - 80 = 0 \]
   \[ (x - 5)^2 = 80 \]
   \[ x = 5 \pm 4\sqrt{5} \]

9. \( x^2 + 12x + 43 = 0 \)
10. \( 4x^2 + 76 = 16x \)

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**Warm-Up**

Graph. What do you notice about the x-intercepts, zeroes, or roots?

- a) \( y = (x - 1)(x + 1) \)
- b) \( y = (x - 1)^2 \)
- c) \( y = x^2 + \frac{1}{4} \)

2 roots

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**The Imaginary Numbers**

- Equations such as \( x^2 + 1 = 0 \) have no real solution, so mathematicians defined the imaginary numbers to represent non-real solutions.
- The imaginary unit \( \textbf{i} \) is defined as \( \sqrt{-1} \). This is useful when working with square roots of negative numbers.
- A pure imaginary number is written in the form \( bi \), where \( b \) is the real number and \( \textbf{i} \) is the imaginary part.
Simplifying Negative Square Roots

1. \( \sqrt{-9} = \sqrt{9 \cdot -1} = 3\sqrt{-1} = 3i \)
2. \( \sqrt{-196} = \sqrt{196 \cdot -1} = 14i \)
3. \( \sqrt{-5} = \sqrt{5 \cdot -1} = \pm \sqrt{5}i \)
4. \( \sqrt{-32} = \sqrt{32 \cdot -1} = \frac{4\sqrt{2}}{4} \sqrt{-1} = \sqrt{2}i \)
5. \( \sqrt{-142} = \sqrt{142 \cdot -1} = \frac{\sqrt{142}}{4} \sqrt{-1} = \frac{\sqrt{142}}{4}i \)
6. \( \sqrt{-192} = \sqrt{192 \cdot -1} = \frac{\sqrt{192}}{4} \sqrt{-1} = \frac{\sqrt{192}}{4}i \)

Solving Equations

7. \( x^2 + 81 = 0 \)
   \[ x = \pm 9i \]
8. \( 2x^2 + 9 = 1 \)
   \[ 2x^2 = -8 \]
   \[ x = \pm 2i \]
9. \( 4x^2 + 15 = -15 \)
   \[ 4x^2 = -30 \]
   \[ x = \pm \sqrt{7.5i} \]
10. \( x^2 + 13 = 1 \)
    \[ x^2 = 12 \]
    \[ x = \pm 2\sqrt{3}i \]
11. \( 3x^2 - 5 = -446 \)
    \[ 3x^2 = -441 \]
    \[ x = \pm 7i \]
12. \( -\frac{2}{3} x^2 - 1 = -17 \)
    \[ -\frac{2}{3} x^2 = -16 \]
    \[ x^2 = 24 \]
    \[ x = \pm 2\sqrt{6}i \]
8. \(2x^2 + 9 = 1\)
   \(2x^2 = -8\)
   \(x^2 = -4\)
   \(\sqrt{x^2} = \pm 4\)
   \(x = \pm 4i\)

9. \(4x^2 + 15 = 0\)
   \(4x^2 = -15\)
   \(x^2 = -\frac{15}{4}\)
   \(\sqrt{x^2} = \sqrt{-\frac{15}{4}}\)
   \(x = \pm \frac{1}{2} \sqrt{15}i\)

10. \(x^2 + 13 = 1\)
    \(x^2 = -12\)
    \(\sqrt{x^2} = \pm \sqrt{12}\)
    \(x = \pm \sqrt{12}i\)
    \(x = \pm 2\sqrt{3}i\)

11. \(3x^2 - 5 = -466\)
    \(3x^2 = -461\)
    \(x^2 = -\frac{461}{3}\)
    \(\sqrt{x^2} = \sqrt{-\frac{461}{3}}\)
    \(x = \pm \frac{1}{3} \sqrt{461}i\)

12. \(-\frac{2}{3}x^2 - 1 = 17\)
    \(-\frac{2}{3}x^2 = 18\)
    \(x^2 = -\frac{27}{2}\)
    \(\sqrt{x^2} = \pm \sqrt{-\frac{27}{2}}\)
    \(x = \pm \frac{3}{2} \sqrt{3}i\)

Complex number scavenger for 20 minutes